Coherent control of stimulated emission inside one-dimensional photonic crystals

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In this paper, the quasinormal mode (QNM) theory is applied to discuss the quantum problem of an atom embedded inside a one-dimensional (1D) photonic band gap (PBG) cavity pumped by two counterpropagating laser beams. The e.m. field is quantized in terms of the QNMs in the 1D PBG and the atom modeled as a two-level system is assumed to be weakly coupled to just one of the QNMs. The main result of the paper is that the decay time depends on the position of the dipole inside the cavity, and can be controlled by the phase difference of the two laser beams.

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I. INTRODUCTION

The behavior of small systems (few degrees of freedom) coupled to dissipative reservoirs represents a central theme in many physical contexts. In quantum optics and in in its simplest form, the problem consists of a two-level excited atom decaying in open space to its ground state through an allowed electric dipole transition. In this condition the emitted radiation propagates away, never coming back to the atom. This is a prototype of irreversible decay of a prepared state, as well as a classical manifestation of the quantization of the electromagnetic field [1].

Spontaneous or stimulated emission is a fundamental process resulting from the interaction between radiation and matter. It depends not only on the properties of the excited atomic system but also on the nature of the environment to which the system is optically coupled [2,3]. It is possible to control the rate of spontaneous or stimulated emission from an excited atom by altering the density of electromagnetic modes near the resonant frequency, i.e., by modifying the accessible modes into which the excited atom can radiate. If the modal density in the vicinity of the frequency of interest is less than that of free space, the atomic decay will be retarded, if it is greater it will be accelerated [4,5].

An important and intriguing situation arose when it was realized that it is possible to create environments in which the spectrum of the electromagnetic field exhibits gaps in frequency. In other words, no radiation over some extended range of frequency can propagate in that environment. Simply stated, an excited atom whose transition frequency falls in the range of that gap should at first sight never decay. The structures exhibiting such frequency gaps are referred to as photonic band gap (PBG) materials or PBG crystals or even photonic crystals [6]. The original idea of light localization is due to John [7] and independently to Yablonovitch [8], who was also the first to construct a material exhibiting a gap in the microwave range.

The photon density of states (DOS) is the fundamental feature that determines the behavior of the atom-field system and characterizes the various types of environments. The form and analytical properties of the DOS dictate the type of approximations permissible in formulating the equations governing the time evolution of the system. When the DOS is a smooth function of frequency over the spectral range of the atomic transition, the rate of spontaneous and stimulated emission is described by Fermi's golden rule. Strong modification in the DOS can be effected by means of photonic crystals. Petro *et al.* [9] have reported on modifications of the emission processes of dye molecules embedded in a three-dimensional solid-state photonic crystal exhibiting a stop band in the visible range. The results are interpreted in terms of redistribution of the photon density of states in the photonic crystal [10]. In Ref. [11], a general semiclassical treatment of radiation rates is developed in an inhomogenous medium. The results agree with those of a fully quantum calculation, and are applied to a simple scalar model of a dipole in a one-dimensional periodic lattice of the Kronig-Penney type. The approach of our paper uses a realistic model for the photonic crystal, as a finite cavity with discontinuities in the refractive index, so our approach improves the results of Ref. [11], obtained by the Kronig-Penney model.

Using numerical methods, Centini *et al.* have studied the propagation of counterpropagating pulses in finite photonic crystals in Ref. [12]. Linear interference and localization effects are shown to combine to either enhance or suppress stimulated emission processes, depending on the initial phase difference between the input pulses. These results are interpreted by viewing the photonic crystal as an open cavity, with a field-dependent, electromagnetic density of modes (DOM) sensitive to initial and boundary conditions. The concept of the DOM has been lacking a precise mathematical definition for a finite-size structure. With the explosive growth in the fabrication of photonic crystals and nanostructures, inherently finite in size, a workable definition becomes imperative. In Ref. [13], D'Aguanno *et al.* give a definition of the DOM based on the Green's function for a generic three-dimensional open cavity filled with a linear, isotropic, dielectric material.

Our paper gives a fully quantum treatment of the dynamics of an atom coupled to two pumps counterpropagating inside a photonic crystal cavity by using quasinormal mode (QNM) formalism, i.e., treating the problem in the framework of open systems. A simple definition is introduced for the density of modes in one-dimensional (1D) open cavities, generalizing the concept of the DOM in free space.

II. QUASINORMAL MODES

The problem of the field description inside an open cavity has been discussed by several authors. In particular Leung *et al.* [14] have introduced the description of the electromagnetic field in a one-side-open and homogeneous cavity in terms of quasinormal modes. Because of the leakage, the QNMs are characterized by complex eigenfrequencies and form an orthogonal basis only inside the cavity, according to a noncanonical metrics.

The QNM treatment can be extended to double-side-open and nonhomogeneous cavities, in particular to one-dimensional photonic band gap (PBG) cavities. The validity of the QNM approach has been discussed by proving the QNM completeness and reconstructing the behavior of the e.m. field in Ref. [15].

An open cavity, viewed as a dissipative system, cannot be quantized [16] unless one considers the bath as part of the total universe in which energy is conserved [17]. Ho *et al.* [18] already made an essential first step toward the application of QNMs to cavity quantum electrodynamics phenomema.

In Ref. [19], the second quantization scheme based on the QNM theory has been extended to 1D PBG cavities. The Feynman propagator is introduced to calculate the decay rate of a dipole inside a 1D PBG, related to the DOM, in the presence of vacuum fluctuations outside the cavity.

In Ref. [20], the second QNM quantization is applied to open cavities, but excited by two counterpropagating pumps. The QNM Hamiltonian is expressed in terms of the QNM operators of annihilation and creation, which describe, respectively, the lowering from the QNM nth to the QNM $(n-1)$th and the raising from the QNM nth to the QNM $(n+1)$th. The QNM commutation relation is not canonical, and it depends on the geometry of the cavity and the phase difference of the two pumps.

In this paper, we apply the quasinormal mode theory to discuss the quantum problem of an atom embedded inside a one-dimensional photonic band gap cavity, which is pumped by two counterpropagating laser beams. The e.m. field is quantized in terms of the QNMs in the 1D PBG and the atom is modeled as a two-level system. In the electric dipole approximation, the atom is assumed to be weakly coupled to just one of the QNMs. As a result of the paper, the decay time depends on the position of the dipole inside the cavity, and can be controlled by the phase difference of the two laser beams. Such a system is relevant for a single-atom, phase-sensitive, optical memory device on the atomic scale.

The paper is organized as follows. In Sec. III, some necessary results of Ref. [20] are stated: the correlation functions of the two counterpropagating laser beams are calculated by simply applying the canonical quantization in terms of the universe modes [17]; and the autocorrelation function of the e.m. field inside the open cavity can be obtained directly by extending the second QNM quantization of Refs. [18,19]. In Sec. IV, the operative definition for the density of the (normal) modes in the universe [20] is generalized in order to define the density of the (quasinormal) modes inside an open cavity, filled with a medium with an inhomogeneous refractive index. The link between the DOM and the phase difference of the two counterpropagating pumps is obtained; the DOM is a sinusoidal function of that phase difference. In Sec. V, the coupling of an excited atom to one QNM is discussed inside a 1D PBG structure which is pumped by two counterpropagating laser beams. The stimulated emission is controlled by the phase difference of the two laser beams: the emission rate is totally suppressed or strongly enhanced depending on that phase difference. Conclusions are given in Sec. VI.

III. SECOND QUANTIZATION INSIDE AN OPEN CAVITY EXCITED BY TWO COUNTERPROPAGATING PUMPS

An open cavity (length L), filled with a medium having a refractive index $n(x)$, is excited by two counterpropagating pumps (phase difference $\Delta\varphi$), one $\hat{E}_\omega^{(\rightarrow)}(x)$ coming from the left side, the other $\hat{E}_\omega^{(\leftarrow)}(x)$ from the right side. We used the pedix ω in order to consider the frequency (Fourier) component of the field operator considered. The two pumps $\hat{E}_\omega^{(\rightarrow)}(x)$ and $\hat{E}_\omega^{(\leftarrow)}(x)$ satisfy some boundary conditions, defined as the incoming wave conditions [15]

$$\partial_x \hat{E}_\omega^{(\rightarrow)}(x) = i\omega\sqrt{\rho_0}\hat{E}_\omega^{(\rightarrow)}(x) \quad \text{for } x \leqslant 0,$$

$$\partial_x \hat{E}_\omega^{(\leftarrow)}(x) = -i\omega\sqrt{\rho_0}\hat{E}_\omega^{(\leftarrow)}(x) \quad \text{for } x \geqslant L, \tag{3.1}$$

where $\rho_0=(n_0/c)^2$ and n_0 is the outside refractive index. Owing to the theorem of equivalence for the e.m. sources [21], the two real pumps in the universe can be substituted by two fictitious electrical currents on the surfaces of the cavity,

$$\hat{J}_0(\omega) = -2\sqrt{\rho_0}\,\partial_x \hat{E}_\omega^{(\rightarrow)}(x)\big|_{x=0} = -2i\rho_0\omega\hat{E}_\omega^{(\rightarrow)}(0),$$

$$\hat{J}_L(\omega) = 2\sqrt{\rho_0}\,\partial_x \hat{E}_\omega^{(\leftarrow)}(x)\big|_{x=L} = -2i\rho_0\omega\hat{E}_\omega^{(\leftarrow)}(L). \tag{3.2}$$

A. Canonical quantization for the two counterpropagating pumps

In the free space, the two counterpropagating pumps are tuned at the frequency ω and prepared in the quantum state $|\psi_0\rangle=|\psi(t=0)\rangle$. The initial state $|\psi_0\rangle$ coincides with a coherent state, i.e., it is an eigenket of the annihilation operator $\hat{a}_\omega$ for the normal modes of the universe [17]. The expectation values of the annihilation $\hat{a}_\omega$ and creation $\hat{a}_\omega^\dagger$ operators must be calculated in the coherent state $|\psi_0\rangle$. As a consequence, the expectation values of the photon number and Hamiltonian operators are, respectively, $\langle\hat{N}_\omega\rangle=\langle\psi_0|\hat{a}_\omega^\dagger\hat{a}_\omega|\psi_0\rangle$ and $\langle\hat{H}_\omega\rangle=\hbar\omega\langle\hat{N}_\omega\rangle$, where $\hbar=h/(2\pi)$, h being Planck's constant.

Let us to adopt the Heisenberg representation in the Fourier domain. The quantum variances of the two fictitious currents (3.2) on the coherent state $|\psi_0\rangle$ are operatively defined as

$$\langle\hat{J}_0^\dagger(\omega)\hat{J}_0(\omega)\rangle = \langle\psi_0|\hat{J}_0^\dagger(\omega)\hat{J}_0(\omega)|\psi_0\rangle,$$

$$\langle\hat{J}_L^\dagger(\omega)\hat{J}_L(\omega)\rangle = \langle\psi_0|\hat{J}_L^\dagger(\omega)\hat{J}_L(\omega)|\psi_0\rangle, \tag{3.3}$$

which can be calculated as

$$\langle \hat{J}_0^\dagger(\omega)\hat{J}_0(\omega)\rangle = \langle \hat{J}_L^\dagger(\omega)\hat{J}_L(\omega)\rangle = \frac{\sqrt{\rho_0}}{\varepsilon_0 n_0^2}(\rho_0\omega)^2\langle \hat{H}_\omega\rangle, \quad (3.4)$$

ε_0 being the dielectric constant in vacuum. If the two surface source current operators are linked to two counterpropagating fields, the quantum cross correlations of these two fictitious operators, calculated on a coherent state of the incoming radiation field, show the properties

$$\langle \hat{J}_L^\dagger(\omega)\hat{J}_0(\omega)\rangle = \langle \hat{J}_0^\dagger(\omega)\hat{J}_L(\omega)\rangle^* = \langle \hat{J}_0^\dagger(\omega)\hat{J}_0(\omega)\rangle \exp(i\Delta\varphi), \quad (3.5)$$

which in a classical view means that the two fields involved are perfectly coherent [21].

B. QNM quantization inside the open cavity

The QNMs are a discrete set of couples $[\omega_n, f_n^N(x)]$ such that $\mathrm{Im}(\omega_n)<0$, $n\in\mathbb{Z}=\{0,\pm1,\pm2,\ldots\}$ and form an orthogonal basis only inside the open cavity according to complex metrics [15]. The e.m. field operator $\hat{E}_\omega(x)$, which in the Fourier domain satisfies the property $\hat{E}_\omega^\dagger(x)=\hat{E}_{-\omega}(x)$, is represented in terms of the QNM operators $\hat{a}_n(\omega)$ in the "(frequency) Heisenberg representation" [18]

$$\hat{E}_\omega(x) = \sum_{n=-\infty}^{\infty} \hat{a}_n(\omega) f_n^N(x), \quad (3.6)$$

and the operators $\hat{a}_n(\omega)$ have to satisfy the property $\hat{a}_n^\dagger(\omega)=\hat{a}_{-n}(-\omega)$, which is similar to the expression valids for canonical operators [16,17]. If the equal-time canonical quantization rule is applied, the QNM commutation relation can be derived in the absence of external pumping [19] or in the presence of two counterpropagating pumps: the QNM commutation relation is not canonical, and it depends on the geometry of the cavity and the phase difference of the two pumps.

The cavity is excited by the two pumps in the coherent state $|\psi_0\rangle$. As a consequence, the expectation values of the QNM operators $\hat{a}_n(\omega)$ can be directly calculated on the same state $|\psi_0\rangle$. Applying the method of Ref. [19], the autocorrelation function of the e.m. field can be calculated as a superposition of the QNMs

$$\begin{aligned} F(x,x',\omega) &= \langle \hat{E}_\omega^\dagger(x)\hat{E}_\omega(x')\rangle = \langle \hat{E}_{-\omega}(x)\hat{E}_\omega(x')\rangle = \langle \hat{J}_0^\dagger(\omega)\hat{J}_0(\omega)\rangle \sum_{n=-\infty}^{\infty}\sum_{n'=-\infty}^{\infty} \frac{f_n^N(0)f_{n'}^N(0)f_n^N(x)f_{n'}^N(x')}{4\rho_0\omega_n\omega_{n'}(\omega_n-\omega)(\omega_{n'}+\omega)} \\ &+ \langle \hat{J}_L^\dagger(\omega)\hat{J}_L(\omega)\rangle \sum_{n=-\infty}^{\infty}\sum_{n'=-\infty}^{\infty} \frac{f_n^N(L)f_{n'}^N(L)f_n^N(x)f_{n'}^N(x')}{4\rho_0\omega_n\omega_{n'}(\omega_n-\omega)(\omega_{n'}+\omega)} + \langle \hat{J}_L^\dagger(\omega)\hat{J}_0(\omega)\rangle \sum_{n=-\infty}^{\infty}\sum_{n'=-\infty}^{\infty} \frac{f_n^N(0)f_{n'}^N(L)f_n^N(x)f_{n'}^N(x')}{4\rho_0\omega_n\omega_{n'}(\omega_n-\omega)(\omega_{n'}+\omega)} \\ &+ \langle \hat{J}_0^\dagger(\omega)\hat{J}_L(\omega)\rangle \sum_{n=-\infty}^{\infty}\sum_{n'=-\infty}^{\infty} \frac{f_n^N(L)f_{n'}^N(0)f_n^N(x)f_{n'}^N(x')}{4\rho_0\omega_n\omega_{n'}(\omega_n-\omega)(\omega_{n'}+\omega)}, \end{aligned} \quad (3.7)$$

where in the previous calculation we used the property $\hat{J}_{0/L}^\dagger(\omega)=\hat{J}_{0/L}(-\omega)$ for the current field operators [20].

IV. DENSITY OF MODES

The quantum state of an e.m. field, excited inside an open cavity, is generally the superposition of infinite eigenstates, which are the quasinormal modes. In this sense, a "density" of "modes" can be introduced as the density of the probability that the e.m. field, excited inside the cavity, is just in one QNM, oscillating in a range of frequency.

As a first step, a local density of modes $\sigma^{(loc)}(x,\omega)$ can be introduced. The local DOM defines the probability $d^2p(x,\omega)$ that an e.m. field, inside an infinitesimal cavity $(x,x+dx)$, is just in one QNM, oscillating in an infinitesimal range of frequency $(\omega,\omega+d\omega)$, i.e.,

$$d^2p(x,\omega) = \sigma^{(loc)}(x,\omega)dx\, d\omega. \quad (4.1)$$

The (integral) density of modes $\sigma(\omega)$ remains to be defined. The (integral) DOM is linked to the probability $dp(\omega)$ that an e.m. field, inside a cavity of length L, is just in one QNM, oscillating in an infinitesimal range of frequency $(\omega,\omega+d\omega)$, i.e.,

$$dp(\omega) = \sigma(\omega)L\, d\omega. \quad (4.2)$$

If the geometry of the cavity is represented by the interval $C=(0,L)$, the (integral) DOM is calculated as the spatial average of the local DOM inside the cavity, i.e.,

$$\sigma(\omega) = \frac{1}{L}\int_0^L \sigma^{(loc)}(x,\omega)dx. \quad (4.3)$$

The universe can be viewed as a cavity of infinite length $L\to\infty$. The e.m. field inside the universe is generally the continuum of the normal modes [17]. The density of probability that the e.m. field excited inside the universe is just in the normal mode tuned at the frequency ω, i.e., $f_\omega(x)=1/\sqrt{2\pi}\exp(i\omega\sqrt{\rho_0}x)$, is obtained as

$$\sigma_{ref}^{(loc)}(x,\omega) = \sqrt{\rho_0}|f_\omega(x)|^2 = \sigma_{ref}(\omega) = \sqrt{\rho_0}/2\pi. \quad (4.4)$$

The (local) DOM for the universe modes is the ratio between the autocorrelation function of the e.m. field in the universe and the expectation value of the e.m energy [20]. By analogy, let us propose an operative definition of the local DOM $\sigma^{(loc)}(x,\omega)$ for the open cavity of relative index $n(x)$, which is pumped by the two counterpropagating laser beams. The local DOM $\sigma^{(loc)}(x,\omega)$ is viewed as an $n^2(x)$-modulated transfer function, if the input is the e.m energy of the two laser beams and the output is the autocorrelation function (3.7) of the e.m. field inside the cavity:

$$\sigma^{(loc)}(x,\omega) \triangleq K\frac{\varepsilon_0 n^2(x)}{\pi}\frac{F(x,x,\omega)}{\langle \hat{H}_\omega\rangle}, \tag{4.5}$$

K being a suitable constant of normalization.

If the open cavity is a symmetric quarter-wave 1D PBG with reference frequency ω_{ref}, number of periods N, and length L, then, over the $[0,2\omega_{ref})$ range $2N+1$ QNMs in units of L can be excited [15]; so the constant K_σ is obtained by the normalization condition

$$\int_0^{2\omega_{ref}} \sigma(\omega)d\omega = \frac{2N+1}{L}. \tag{4.6}$$

DOM and phase difference of the two counterpropagating laser beams

Inside the cavity, if the condition of QNM completeness is applied [14],

$$\sum_{n=-\infty}^{\infty}\frac{f_n^N(x)f_n^N(x')}{\omega_n}=0 \quad \forall\ x,x' \in (0,L), \tag{4.7}$$

the autocorrelation function of the e.m. field (3.7) reduces to

$$\begin{aligned}F(x,x',\omega) = \frac{\rho_0^{3/2}}{4\varepsilon_0 n_0^2}\langle\hat{H}_\omega\rangle\Bigg[&\sum_{n=-\infty}^{\infty}\sum_{n'=-\infty}^{\infty}\frac{f_n^N(0)f_{n'}^N(0)+f_n^N(L)f_{n'}^N(L)}{(\omega-\omega_n)(\omega+\omega_{n'})}f_n^N(x)f_{n'}^N(x')\\ &+\sum_{n=-\infty}^{\infty}\sum_{n'=-\infty}^{\infty}\frac{f_n^N(0)f_{n'}^N(L)\exp(i\Delta\varphi)+f_n^N(L)f_{n'}^N(0)\exp(-i\Delta\varphi)}{(\omega-\omega_n)(\omega+\omega_{n'})}f_n^N(x)f_{n'}^N(x')\Bigg],\end{aligned} \tag{4.8}$$

and the local DOM can be obtained as

$$\begin{aligned}\sigma^{(loc)}(x,\omega) = K\frac{\sqrt{\rho_0}}{4\pi}\rho(x)\Bigg[&\sum_{n=-\infty}^{\infty}\sum_{n'=-\infty}^{\infty}\frac{f_n^N(0)f_{n'}^N(0)+f_n^N(L)f_{n'}^N(L)}{(\omega-\omega_n)(\omega+\omega_{n'})}f_n^N(x)f_{n'}^N(x)\\ &+\sum_{n=-\infty}^{\infty}\sum_{n'=-\infty}^{\infty}\frac{f_n^N(0)f_{n'}^N(L)\exp(i\Delta\varphi)+f_n^N(L)f_{n'}^N(0)\exp(-i\Delta\varphi)}{(\omega-\omega_n)(\omega+\omega_{n'})}f_n^N(x)f_{n'}^N(x)\Bigg].\end{aligned} \tag{4.9}$$

When the e.m. field consists of one QNM ($n=-n$ [14]), the local DOM $\sigma_n^{(loc)}(x,\omega)$ can be related to the (integral) DOM (see Ref. [19]) as

$$\begin{aligned}\sigma_n(\omega) = \frac{1}{L}\int_0^L \sigma_n^{(loc)}(x,\omega)dx = K\frac{\sqrt{\rho_0}}{4\pi}\frac{I_n}{(\omega-\mathrm{Re}\ \omega_n)^2+\mathrm{Im}^2\ \omega_n}&\{|f_n^N(0)|^2+|f_n^N(L)|^2+f_n^N(0)[f_n^N(L)]^*\\ &\times\exp(i\Delta\varphi)+f_n^N(L)[f_n^N(0)]^*\exp(-i\Delta\varphi)\}\end{aligned} \tag{4.10}$$

where I_n are normalization integrals (see Ref. [19]).

In the conservative limit, the open cavity is characterized by narrow resonances $|\mathrm{Im}(\omega_n)| \ll |\mathrm{Re}(\omega_n)|$, so each normalization integral is $I_n \approx 1/L$ and the total DOM is the superposition of all the DOMs (4.10), i.e.,

$$\sigma(\omega) \cong \sum_{n=-\infty}^{\infty}\sigma_n(\omega) \cong K\frac{\sqrt{\rho_0}}{4\pi L}\sum_{n=-\infty}^{\infty}\frac{|f_n^N(0)|^2+|f_n^N(L)|^2}{(\omega-\mathrm{Re}\ \omega_n)^2+\mathrm{Im}^2\ \omega_n}+K\frac{\sqrt{\rho_0}}{4\pi L}\sum_{n=-\infty}^{\infty}\frac{f_n^N(0)[f_n^N(L)]^*\exp(i\Delta\varphi)+f_n^N(L)[f_n^N(0)]^*\exp(-i\Delta\varphi)}{(\omega-\mathrm{Re}\ \omega_n)^2+\mathrm{Im}^2\ \omega_n}. \tag{4.11}$$

If the cavity presents a refractive index $n(x)$ which satisfies the symmetry properties $n(L/2-x)=n(L/2+x)$, on the surfaces of the cavity the values of the QNM functions are such that $f_n^N(L)=(-1)^n f_n^N(0)$. If $I_n \approx 1/L$, then $|f_n^N(0)|^2 \cong |\mathrm{Im}\ \omega_n|/\sqrt{\rho_0}$ [19]. The DOM (4.11) can thus be settled as

$$\sigma(\omega) = K[\tilde{\sigma}_1(\omega) + \tilde{\sigma}_2(\omega)\cos(\Delta\varphi)], \quad (4.12)$$

where

$$\tilde{\sigma}_1(\omega) = \frac{1}{2\pi L}\sum_{n=-\infty}^{\infty} \frac{|\mathrm{Im}\ \omega_n|}{(\omega - \mathrm{Re}\ \omega_n)^2 + \mathrm{Im}^2\ \omega_n}, \quad (4.13)$$

$$\tilde{\sigma}_2(\omega) = \frac{1}{2\pi L}\sum_{n=-\infty}^{\infty} (-1)^n \frac{|\mathrm{Im}\ \omega_n|}{(\omega - \mathrm{Re}\ \omega_n)^2 + \mathrm{Im}^2\ \omega_n}. \quad (4.14)$$

Equation (4.13) can be interpreted as the DOM due to only one laser beam; it is a series of infinite QNM Lorentzian functions and coincides with the DOM due to the vacuum fluctuations [19]: in fact, the potential number of QNMs inside the cavity is independent of the statistics of the pumping because it is fixed only by the geometry of the cavity, which acts as an e.m. filter of frequency; moreover, the density of the probability to excite one QNM (i.e., the DOM) cannot be modified because the vacuum fluctuations do not add other degrees of freedom in the universe. Equation (4.14) can be interpreted as the interference term due to the two counterpropagating laser beams and it is an alternate-sign series of the same Lorentzian functions. The interference of the two laser beams produces the control of the DOM inside the open cavity. In fact, the phase difference adds one degree of freedom in the universe; so, if the two counterpropagating laser beams are in phase $\Delta\varphi=0$, the DOM is a series of even QNM Lorentzian functions, while if the two laser beams are opposite in phase $\Delta\varphi=\pi$, the DOM is a series of odd Lorentzian functions. This result puts well into evidence how the DOM of an open cavity depends on the excitation condition of the cavity.

A quarter-wave symmetric 1D PBG, with N periods and ω_{ref} as reference frequency, presents $2N+1$ families of QNMs [15], i.e., $m=0,1,\ldots,2N$: the mth family of QNMs consists of infinite QNM frequencies $\omega_{m,k}, k \in \mathbb{Z} =\{0,\pm 1,\pm 2,\ldots\}$, with the same imaginary part $\mathrm{Im}(\omega_{m,k}) =\mathrm{Im}(\omega_{m,0})$ and lined by a step $\Delta=2\omega_{ref}$, i.e., $\mathrm{Re}(\omega_{m,k}) =\mathrm{Re}(\omega_{m,0})+k\Delta$. The DOM (4.13) can be specified as [15]

$$\tilde{\sigma}_1(\omega) = \frac{1}{4L\Delta}\sum_{m=0}^{2N} \coth\left[i\frac{\pi}{\Delta}(\omega - \omega^*_{m,0})\right] + \frac{i}{4L\Delta}\sum_{m=0}^{2N} \cot\left[\frac{\pi}{\Delta}(\omega - \omega_{m,0})\right], \quad (4.15)$$

while the DOM (4.14) converges to

$$\tilde{\sigma}_2(\omega) = \frac{1}{2\pi L}\sum_{m=0}^{2N}\sum_{k=-\infty}^{\infty} (-1)^{m+k} \times \frac{|\mathrm{Im}\ \omega_{m,0}|}{[\omega - (\mathrm{Re}\ \omega_{m,0} + k\Delta)]^2 + \mathrm{Im}^2\ \omega_{m,0}} = \frac{i}{4L\Delta}\sum_{m=0}^{2N} (-1)^m \csc\left[\frac{\pi}{\Delta}(\omega - \omega_{m,0})\right] - \frac{i}{4L\Delta}\sum_{m=0}^{2N} (-1)^m \csc\left[\frac{\pi}{\Delta}(\omega - \omega^*_{m,0})\right]. \quad (4.16)$$

The number of QNMs over the $[0,\Delta)$ range is $2N+1$ in units of L. The constant K in the DOM (4.12) can be determined by the condition of normalization (4.6), i.e.,

$$K = \frac{2N+1}{L}\frac{1}{S}, \quad (4.17)$$

where

$$S = S_1 + S_2\cos(\Delta\varphi), \quad S_1 = \int_0^{\Delta} \tilde{\sigma}_1(\omega)d\omega,$$

$$S_2 = \int_0^{\Delta} \tilde{\sigma}_2(\omega)d\omega. \quad (4.18)$$

In Fig. 1, the density of modes (4.12) is plotted, in units of the reference DOM (4.4), as a function of the dimensionless frequency ω/ω_{ref}, for three different quarter-wave symmetric 1D PBGs: (a) $\lambda_{ref}=1\ \mu$m, $N=6$, $n_h=2$, $n_l=1.5$; (b) $\lambda_{ref}=1\ \mu$m, $N=6$, $n_h=3$ $n_l=2$; (c) $\lambda_{ref}=1\ \mu$m, $N=7$, $n_h=3$ $n_l=2$. In each figure, the DOM is plotted for an external pumping consisting of one laser beam or the vacuum fluctuations (dotted line) [see Eq. (4.15)]; two counterpropagating laser beams in phase $\Delta\varphi=0$ (continuous thin line) [see Eq. (4.16)]; and two laser beams opposite in phase $\Delta\varphi=\pi$ (continuous thick line). A quarter-wave symmetric 1D PBG, with N periods and ω_{ref} as reference frequency, presents $2N+1$ QNMs in the $[0,2\omega_{ref})$ range., i.e., $k=0,1,\ldots,2N$ excluding $\omega=2\omega_{ref}$. If the 1D PBG cavity is excited by two counterpropagating laser beams with a phase difference $\Delta\varphi$, the even (odd) QNMs, i.e., $k=0,2,\ldots,2N$ ($k=1,3,\ldots,2N-1$), increase in strength when the two laser beams are in phase, i.e., $\Delta\varphi=0$ (opposite in phase, i.e., $\Delta\varphi=\pi$), and almost flag when the two laser beams are opposite in phase, i.e., $\Delta\varphi=\pi$ (in phase, i.e., $\Delta\varphi=0$) [see Figs. 1(a) and 1(b)]. If one period is added to the 1D PBG, the QNMs next to the low and high frequency band edges exchange their physical response to the phase difference of the two laser beams [see Fig. 1(c)]. The density of modes is the physical key to discussing the coherent control of the stimulated emission.

V. ATOMIC EMISSION POWER

In order to discuss the emission processes of an atom embedded inside an open cavity excited by two counterpropagating pumps, let us consider the atom as a two-level system and the e.m. field in the cavity as a superposition of quasinormal modes. The dipole approximation [16] is assumed, so the two-level system acts as an electrical dipole of length l, placed in a point, x_0, which oscillates orthogonally to the x direction. At the initial time $t=0$, the dipole is prepared in the excited state corresponding to the higher energy, and its momentum $\hat{P}$ has the mean value $\langle\hat{P}\rangle_{t=0}$. The dipole bandwidth is so narrow, if compared with the QNM spectrum [18], that the dipole is assumed to be (weakly) coupled with

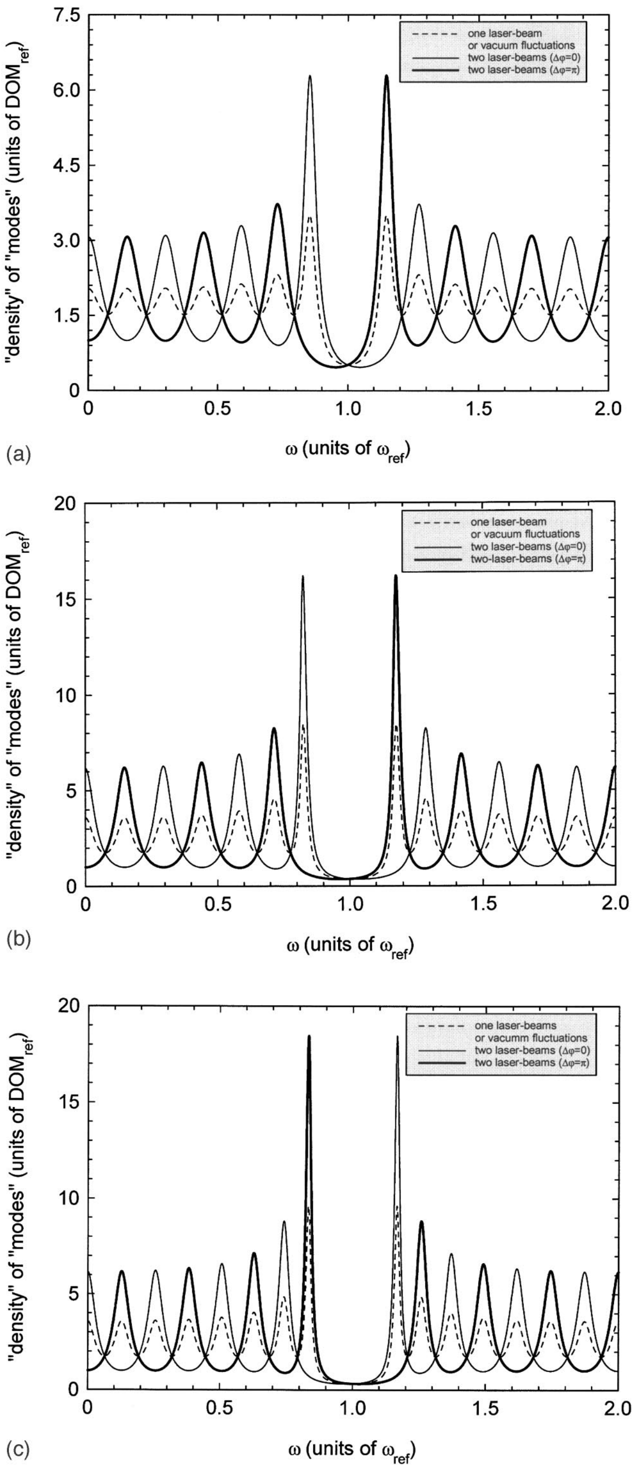

FIG. 1. The density of modes [Eq. (4.12)] is plotted, in units of the reference DOM [Eq. (4.4)] as a function of the dimensionless frequency ω/ω_{ref}, for three different quarter-wave symmetric 1D PBGs: (a) $\lambda_{ref}=1$ μm, $N=6$, $n_h=2$, $n_l=1.5$; (b) $\lambda_{ref}=1$ μm, $N=6$, $n_h=3$, $n_l=2$; (c) $\lambda_{ref}=1$ μm, $N=7$, $n_h=3$, $n_l=2$. In each figure, the DOM is plotted for an external pumping consisting of one laser beam or the vacuum fluctuations (dotted line) [see Eq. (4.15)]; two counterpropagating laser beams in phase $\Delta\varphi=0$ (continuous thin line) [see Eq. (4.16)]; two laser beams of opposite phase $\Delta\varphi=\pi$ (continuous thick line).

just one of the QNMs. The cavity is filled with a medium of refractive index $n(x)$, the two pumps are in the coherent state $|\psi_0\rangle$, and the dipole, coupled with one QNM, is excited with a local density of probability (DOM) $\sigma^{(loc)}(x,\omega)$ [see Eqs. (4.5) and (4.6)].

At the time $t>0$, when the dipole is in the state $|\psi(t)\rangle$ [17], the e.m. power $\overline{\langle\hat{W}\rangle_t}$, supplied by the dipole inside the open cavity [22,23], is inversely proportional to the relative dielectric constant $n^2(x_0)$ and is directly linked to the local DOM $\sigma^{(loc)}(x,\omega)$, i.e., [13]

$$\overline{\langle\hat{W}\rangle_t} = -\frac{\pi/2}{\varepsilon_0 n^2(x_0)}(\langle\hat{P}\rangle_{t=0}/l)^2\omega^2\sigma^{(loc)}(x_0,\omega). \qquad (5.1)$$

If the cavity is homogeneous and extends to the whole universe, then the refractive index $n(x_0)=n_0$ and the DOM $\sigma_{ref}(x,\omega)=\sigma_{ref}(\omega)=\sqrt{\rho_0}/2\pi$ become uniform. The time average for the mean value of the e.m. power (5.1), supplied by the dipole free in the universe, reduces to

$$\overline{\langle\hat{W}\rangle_t} = -\frac{\pi/2}{\varepsilon_0 n_0^2}(\langle\hat{P}\rangle_{t=0}/l)^2\omega^2\sigma_{ref}(\omega) = -\frac{\sqrt{\rho_0}}{4\varepsilon_0 n_0^2}(\langle\hat{P}\rangle_{t=0}/l)^2\omega^2. \qquad (5.2)$$

If the two counterpropagating pumps are filtered at $\omega \approx \mathrm{Re}\,\omega_n$, just the nth QNM is excited and not the other QNMs, because the nth QNM oscillates at resonance frequency $\omega\approx\mathrm{Re}\,\omega_n$ within the narrow range $2|\mathrm{Im}\,\omega_n| \ll |\mathrm{Re}\,\omega_n|$, so distant enough from the other QNMs [15]. The e.m. power $\overline{\langle\hat{W}_n\rangle_t}$ supplied by the dipole to the nth QNM, i.e.,

$$\overline{\langle\hat{W}_n\rangle_t} = -\frac{\pi/2}{\varepsilon_0 n^2(x_0)}(\langle\hat{P}\rangle_{t=0}/l)^2\omega^2\sigma_n^{(loc)}(x_0,\omega), \qquad (5.3)$$

is proportional to the local density of probability (DOM) for the nth QNM [19], i.e.,

$$\sigma_n^{(loc)}(x_0,\omega) = \frac{1}{I_n}\sigma_n(\omega)\rho(x_0)|f_n^N(x_0)|^2, \qquad (5.4)$$

which is directly linked to the (integral) DOM for the nth QNM, i.e., $\sigma_n(\omega)$.

In order to discuss the processes of spontaneous emission, consider the two pumps in the ground state of the e.m. field [17]; the DOM for the nth QNM is expressed as [19]

$$\sigma_n^{(A)}(\omega) = K_n\frac{L}{2\pi}\frac{I_n^2|\mathrm{Im}\,\omega_n|}{(\omega-\mathrm{Re}\,\omega_n)^2+\mathrm{Im}^2\,\omega_n}. \qquad (5.5)$$

The normalization constant K_n can be obtained by the following condition:

$$\int_{\mathrm{Re}\,\omega_n-|\mathrm{Im}\,\omega_n|}^{\mathrm{Re}\,\omega_n+|\mathrm{Im}\,\omega_n|}\sigma_n^{(A)}(\omega)d\omega = \frac{1}{L}. \qquad (5.6)$$

In order to discuss the processes of stimulated emission, consider the two pumps in the coherent state $|\psi_0\rangle$; when the cavity presents a refractive index $n(x)$ which satisfies the symmetry properties $n(L/2-x)==n(L/2+x)$, the DOM for the nth QNM can be simplified as (see Sec. IV)

$$\sigma_n^{(B)}(\omega) = \sigma_n^{(A)}(\omega)[1 + (-1)^n \cos \Delta\varphi]. \tag{5.7}$$

As from Eq. (5.7), the density of probability to excite the dipole coupled with the nth QNM can be controlled by the phase difference $\Delta\varphi$ of the two external pumps.

A. Dipole-QNM coupling: Decay time in units of dwell time

Let us define the *sensitivity function* $S(x_0,\omega)$ of a *dipole-cavity coupling* as the ratio between the e.m. power (5.1) supplied by a dipole located inside an open cavity and the e.m. power (5.2) supplied by the same dipole free in the universe:

$$S(x_0,\omega) \triangleq \frac{\overline{\langle \hat{W} \rangle_t}}{\langle \hat{W}_{ref} \rangle_t}. \tag{5.8}$$

Inserting Eqs. (5.1) and (5.2) in Eq. (5.8), it results that

$$S(x_0,\omega) = \frac{\sigma^{(loc)}(x_0,\omega)/\sigma_{ref}(\omega)}{[n(x_0)/n_0]^2}. \tag{5.9}$$

If the relative dielectric constant is inhomogeneous $n_0^2 \to n^2(x_0)$, the local DOM is modified $\sigma_{ref}(\omega) \to \sigma^{(loc)}(x_0,\omega)$; the sensitivity function $S(x_0,\omega)$ can be expressed as the per cent variation of the local DOM $\sigma^{(loc)}(x_0,\omega)/\sigma_{ref}(\omega)$ in units of the per cent variation of the relative dielectric constant $n^2(x_0)/n_0^2$. The meaning of the sensitivity function follows: Eq. (5.9) provides a physical tool to discuss the emission processes of a dipole excited inside a cavity with respect to the case of the same dipole free in the universe.

If the sensitivity function $S_n(x_0,\omega)$ of the dipole–nth QNM coupling, i.e.,

$$S_n(x_0,\omega) = \left[\frac{n_0}{n(x_0)}\right]^2 \frac{\sigma_n^{(loc)}(x_0,\omega)}{\sigma_{ref}(\omega)}, \tag{5.10}$$

is developed by inserting the local DOM of the nth QNM (5.4), it follows that [recalling $\rho(x) = \mathbf{(}n(x)/c\mathbf{)}^2$ and $\rho_0 = (n_0/c)^2$]

$$S_n(x_0,\omega) = \rho_0 \frac{|f_n^N(x_0)|^2}{I_n} \frac{\sigma_n(\omega)}{\sigma_{ref}(\omega)}. \tag{5.11}$$

If the dipole inside the cavity (point x_0) is coupled to the nth QNM (resonance frequency $\omega \approx \mathrm{Re}\ \omega_n$), then the sensitivity function $S_n(x_0,\omega)$ is linked both to the normalized intensity $|f_n^N|^2$ of the nth QNM, sampled in the point x_0, and to the DOM σ_n in the resonance $\omega \approx \mathrm{Re}\ \omega_n$. So the weak coupling of a dipole to one QNM responds to Fermi's golden rule [17].

In order to discuss the processes of spontaneous emission (case A), let us specify the sensitivity function (5.11) in terms of the DOM (5.5), when an open cavity is excited by vacuum fluctuations:

$$\begin{aligned} S_n^{(A)}(x_0,\omega) &= \rho_0 \frac{|f_n^N(x_0)|^2}{I_n} \frac{\sigma_n^{(A)}(\omega)}{\sigma_{ref}(\omega)} \\ &= K_n \sqrt{\rho_0}(LI_n)|f_n^N(x_0)|^2 \frac{|\mathrm{Im}\ \omega_n|}{(\omega - \mathrm{Re}\ \omega_n)^2 + \mathrm{Im}^2\ \omega_n}. \end{aligned} \tag{5.12}$$

As discussed in Ref. [19], for a quarter-wave symmetric 1D PBG with ω_{ref} as reference wavelength and N periods, the $[0,2\omega_{ref})$ range includes $2N+1$ QNMs, which are identified as $|n\rangle$, $n \in [0,N]$. If the dipole is located in the center $x_0 = L/2$ of the 1D PBG, it can be coupled to one of the QNMs with an even n: in fact, in $x_0 = L/2$, the QNM intensity $|f_n^N|^2$ has a maximum for even values of n and is almost null for odd values of n.

The spontaneous emission process of a dipole localized on the surface of the cavity $x_0 = 0$ is characterized by a decay time

$$\tau_n^{(A)}(x_0 = 0) = \frac{1}{\Delta\omega_n^{(A)}(x_0 = 0)}, \tag{5.13}$$

$\Delta\omega_n^{(A)}(x_0 = 0)$ being the bandwidth of the sensitivity function $S_n^{(A)}(x_0 = 0,\omega)$ at half height $[S_n^{(A)}(0,\omega = \mathrm{Re}\ \omega_n)/2]$. After some algebra, recalling $|f_n^N(0)|^2 = |\mathrm{Im}\ \omega_n|/\sqrt{\rho_0}$, it results that

$$\tau_n^{(A)}(x_0 = 0) = \frac{1}{2|\mathrm{Im}\ \omega_n|}. \tag{5.14}$$

It is clear that no emission occurs $[\tau_n^{(A)}(x_0 = 0) \to \infty]$ when the cavity is closed $[|\mathrm{Im}\ \omega_n| \to 0]$.

The spontaneous emission process of a dipole embedded in the point x_0 of the cavity is characterized by a decay time

$$\tau_n^{(A)}(x_0) = \frac{1}{\Delta\omega_n^{(A)}(x_0)}, \tag{5.15}$$

$\Delta\omega_n^{(A)}(x_0)$ being the bandwidth of the sensitivity function $S_n^{(A)}(x_0,\omega)$ at the half height of $S_n^{(A)}(x_0 = 0,\omega)$. After some algebra, it results that

$$[\tau_n^{(A)}(x_0)]^2 \cong \frac{1}{4}\left(\frac{LI_n}{\sqrt{\rho_0}}\right) \frac{\tau_n^{(A)}(x_0 = 0)}{|f_n^N(x_0)|^2}. \tag{5.16}$$

If the cavity of length L is pumped by vacuum fluctuations filtered at $\omega \approx \mathrm{Re}\ \omega_n$, it is possible to introduce the dwell time [13] of vacuum fluctuations, linked to the DOM (5.5):

$$\begin{aligned} \delta t_n^{(A)}(\omega) &\triangleq L\sigma_n^{(A)}(\omega) = K_n \frac{(LI_n)^2}{2\pi} \frac{|\mathrm{Im}\ \omega_n|}{(\omega - \mathrm{Re}\ \omega_n)^2 + \mathrm{Im}^2\ \omega_n} \\ &\cong \frac{K_n}{\pi} \frac{(LI_n)^2}{2|\mathrm{Im}\ \omega_n|}. \end{aligned} \tag{5.17}$$

The decay time (5.14) of the dipole coupled with the nth QNM, when the dipole is on the surface $x_0 = 0$, is different from the dwell time (5.17) of vacuum fluctuations at $\omega \approx \mathrm{Re}\ \omega_n$; in fact, if the dipole is on one surface of a quarter-wave symmetric 1D PBG with parameters $\lambda_{ref} = 1\ \mu\mathrm{m}$, $N = 6$, $n_h = 3$, $n_l = 2$ and it is excited by vacuum fluctuations filtered at one band edge $\omega_{I\ band\ edge}/\omega_{ref} \approx 0.8249$ or

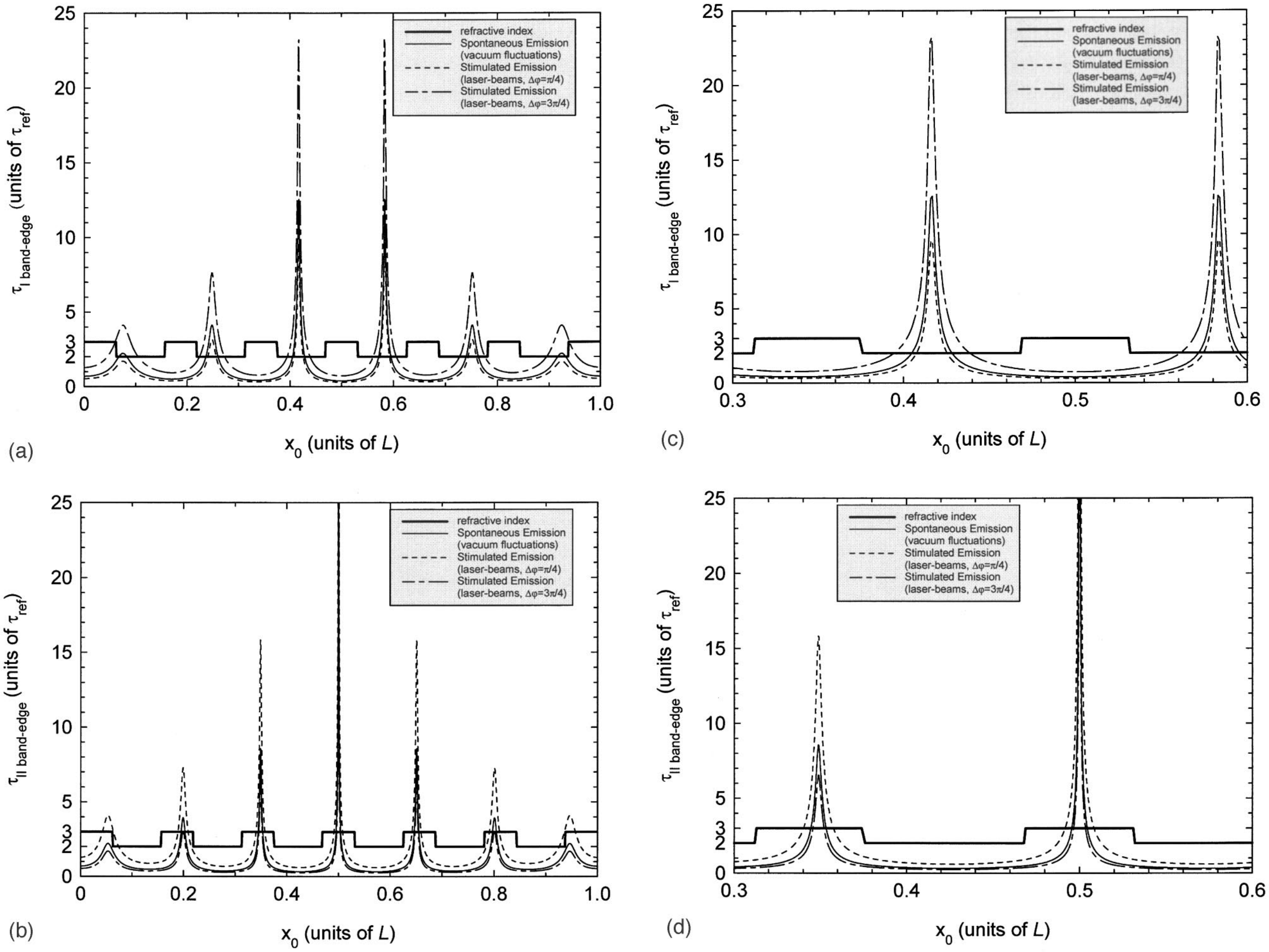

FIG. 2. An excited dipole is embedded inside a quarter-wave symmetric 1D PBG with parameters $\lambda_{ref}=1\ \mu$m, $N=6$, $n_h=3$, $n_l=2$, and it is pumped by two counterpropagating waves filtered at the low frequency band edge $\omega_{I\ band\ edge}/\omega_{ref}\approx 0.8249$ (a) or at the high frequency band-edge $\omega_{II\ bandedge}/\omega_{ref}\approx 1.175$ (b). The decay time τ, in units of the decay time τ_{ref} for spontaneous emission when the dipole is on the surface $x_0=0$ of the 1D PBG [see Eq. (5.14)], is plotted as a function of the dimensionless position x_0/L of the dipole, L being the length of the cavity. Several cases are shown: the spontaneous emission [see Eq. (5.16)], when the two pumps describe vacuum fluctuations (——); and the stimulated emission [see Eq. (5.20)], when the two pumps describe two input laser beams, almost in phase (– – –) or of opposite phase (— – —). (c) and (d) are magnifications of (a) and (b), respectively for the decay time in the third and fourth periods of the cavity.

$\omega_{II\ band\ edge}/\omega_{ref}\approx 1.175$, then the decay time is $\omega_{ref}\tau^{(A)}_{band\ edge}(x_0=0)\cong 33.30$ and the ratio between the decay time and the dwell time is $\tau^{(A)}_{band\ edge}(x_0=0)/\delta t^{(A)}_{band\ edge}\cong 1.571$.

In order to discuss the processes of stimulated emission (case B), let us specify the sensitivity function (5.11) in terms of the DOM (5.7), when a symmetric cavity [refractive index $n(x)$ such that $n(L/2-x)=n(L/2+x)$] is excited by two counterpropagating laser beams (phase difference $\Delta\varphi$)

$$\begin{aligned} S_n^{(B)}(x_0,\omega) &= \rho_0 \frac{|f_n^N(x_0)|^2}{I_n}\frac{\sigma_n^{(B)}(\omega)}{\sigma_{ref}(\omega)} \\ &= \rho_0 \frac{|f_n^N(x_0)|^2}{I_n}\frac{\sigma_n^{(A)}(\omega)[1+(-1)^n\cos\Delta\varphi]}{\sigma_{ref}(\omega)} \\ &= S_n^{(A)}(x_0,\omega)[1+(-1)^n\cos\Delta\varphi]. \end{aligned} \tag{5.18}$$

If the dipole is embedded in a point x_0 of the cavity in which the normalized intensity of the nth QNM is almost null, i.e., $|f_n^N(x_0)|^2\cong 0$, all the emission processes are inhibited, i.e., $S_n^{(A)}(x_0,\omega)=S_n^{(B)}(x_0,\omega)\cong 0$. Otherwise, if the dipole is in a point x_0 of the cavity in which the nth QNM intensity is not null, i.e., $|f_n^N(x_0)|^2\neq 0$, it can be coupled to one of the QNMs with an even n, when the two laser beams are in phase $\Delta\varphi=0$, while it can be coupled to one of the QNMs with an odd n, when the two laser beams are opposite in phase $\Delta\varphi=\pi$.

The stimulated emission process of a dipole embedded in the point x_0 of the cavity is characterized by a decay time

$$\tau_n^{(B)}(x_0)=\frac{1}{\Delta\omega_n^{(B)}(x_0)}, \tag{5.19}$$

$\Delta\omega_n^{(B)}(x_0)$ being the bandwidth of the sensitivity function $S_n^{(B)}(x_0,\omega)$ at the half height of $S_n^{(A)}(x_0=0,\omega)$. After some algebra, it results that

$$\tau_n^{(B)}(x_0) \cong \frac{\tau_n^{(A)}(x_0)}{\sqrt{1+(-1)^n \cos \Delta\varphi}}. \tag{5.20}$$

All the bandwidths $\Delta\omega_n^{(A)}(x_0=0)$, $\Delta\omega_n^{(A)}(x_0)$, and $\Delta\omega_n^{(B)}(x_0)$ are referred to the half length of $S_n^{(A)}(x_0=0,\omega)$, so the decay times (5.14), (5.16), and (5.20) run from the same instant; the decay time of the dipole depends on the position of the dipole inside the cavity, and can be controlled by the phase difference of the two laser beams.

Figure 2 refers to an excited dipole, embedded inside a quarter-wave symmetric 1D PBG with parameters $\lambda_{ref}=1$ μm, $N=6$, $n_h=3$, $n_l=2$, pumped by two counterpropagating waves filtered at the low frequency band edge $\omega_{I\ band\ edge}/\omega_{ref}\approx 0.8249$ [Fig. 2(a)] or at the high frequency band edge $\omega_{II\ band\ edge}/\omega_{ref}\approx 1.175$ [Fig. 2(b)]. The decay time τ, in units of the decay time τ_{ref} for spontaneous emission when the dipole is on the surface $x_0=0$ of the 1D PBG [see Eq. (5.14)], is plotted as a function of the dimensionless position x_0/L of the dipole, L being the length of the cavity. Several cases are shown: the spontaneous emission [see Eq. (5.16)], when the two pumps describe vacuum fluctuations (solid line); and the stimulated emission [see Eq. (5.20)], when the two pumps describe two input laser beams, almost in phase (dashed line) or opposite in phase (long-dashed short-dashed line). So, in the low frequency (high frequency) band edge, all the emission processes are enhanced if the dipole is inside the layers with high (low) refractive index; while the stimulated emission can be inhibited by increasing (reducing) the phase difference of the two laser beams if the dipole is inside the layers with low (high) refractive index. In fact, next to the low frequency (high frequency) band edge, the DOM is minimum if the two laser beams are opposite in phase (in phase) [see Fig. 1(b)]. Figure 2(c) [Fig. 2(d)] is a magnification of Fig. 2(a) [Fig. 2(b)] for the decay time when the dipole is in the third and fourth periods of the 1D PBG. If the dipole is centered in the cavity, the decay time is accelerated (tends to be highly retarded) in the low frequency (high frequency) band edge. In fact, in the center of the cavity, the QNM corresponding to the low frequency (high frequency) band edge is maximum (tends to zero) [see comments on Eq. (5.12)].

If the cavity of length L is pumped by the two laser beams tuned at the frequency $\omega\approx \mathrm{Re}\,\omega_n$ (phase difference $\Delta\varphi$), it is possible to introduce the dwell time [13] of the two laser beams, linked to the DOM (5.7) [see Eq. (5.17)]

$$\begin{aligned}\delta t_n^{(B)} &\triangleq L\sigma_n^{(B)} = L\sigma_n^{(A)}[1+(-1)^n\cos\Delta\varphi] \\ &= \delta t_n^{(A)}[1+(-1)^n\cos\Delta\varphi].\end{aligned} \tag{5.21}$$

The decay time (5.20) for stimulated emission of the dipole, coupled with the nth QNM, and the dwell time (5.21) of the two laser beams, tuned at the nth transmission resonance, are dual functions; in fact, the stimulated emission is inhibited [$\tau_n^{(B)}(x_0)\to\infty$] when the two laser beams are reflected by the cavity ($\delta t_n^{(B)}=0$), and it is enhanced [$\tau_n^{(B)}(x_0)$ is minimum] when the two laser beams "stand" in the cavity [$\delta t_n^{(B)}$ is maximum].

Figure 3 refers to the excited dipole embedded inside the quarter-wave symmetric 1D PBG of Fig. 2, pumped by two counterpropagating laser beams filtered at the low [Fig. 3(a)] or at the high frequency band edge [Fig. 3(b)]. The decay time for stimulated emission (solid line), in units of the decay time for spontaneous emission [see Eq. (5.20)], is compared with the dwell time (dashed line) for the two laser beams, in units of the dwell time for vacuum fluctuations [see Eq. (5.21)]. The decay time for stimulated emission and the dwell time for the two laser beams are compared on different scales as functions of the phase difference between the two laser beams. In the low frequency (high frequency) band edge, the decay time ratio is rising (slopes down) and so the dwell time ratio slopes down (is rising) when the phase difference of the two laser beams increases from $\Delta\varphi=0$ to π; in fact, the decay time [see Figs. 2(a) and 2(b)] tends to the maximum and the DOM [see Fig. 1(b)] tends to the minimum when the laser beams are opposite in phase. Then, in the low frequency (high frequency) band edge, the decay time ratio tends to infinity and so the dwell time ratio is null when the phase difference of the two laser beams is

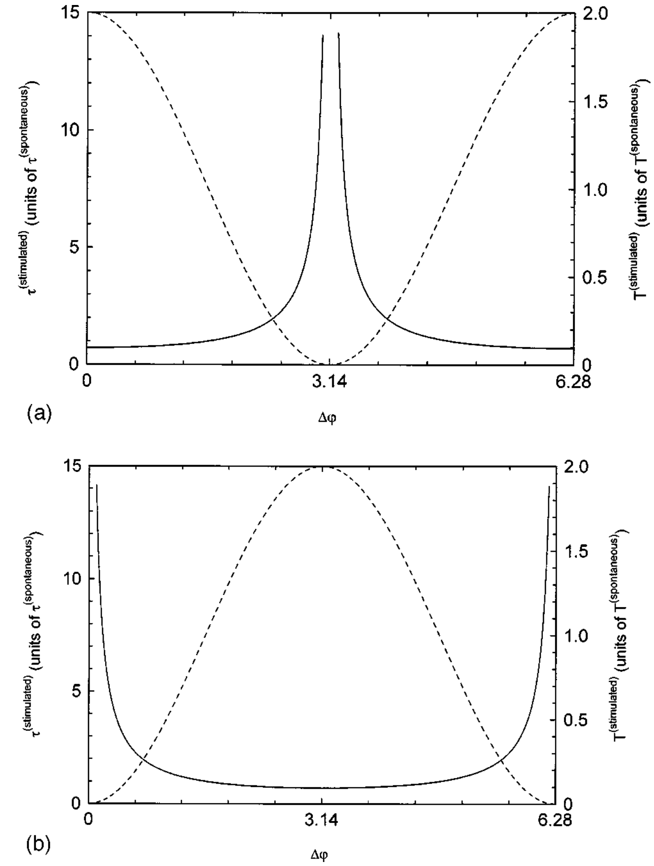

FIG. 3. The excited dipole, embedded inside the quarter-wave symmetric 1D PBG of Fig. 2, is pumped by two counterpropagating laser beams filtered at the low frequency band edge (a) or at the high frequency band edge (b). The decay time for stimulated emission (——), in units of the decay time for spontaneous emission [see Eq. (5.20)], is compared with the dwell time (– – –) for the two laser beams, in units of the dwell time for vacuum fluctuations [see Eq. (5.21)]. The decay time for stimulated emission and the dwell time for the two laser beams are compared on different scales as functions of the phase difference between the two laser beams.

$\Delta\varphi=\pi$ ($\Delta\varphi=0$); in fact, the dipole is not coupled to the QNM corresponding to the low frequency (high frequency) band edge when the two laser beams are (opposite in phase) in phase [see comments on Eq. (5.18)].

VI. CONCLUSIONS

In this paper, we have considered 1D PBG cavities, which present both sides open to the external environment, with a stratified material inside. A 1D PBG is finite in space and, working with electromagnetic pulses of a spatial extension longer than the length of the open cavity, cannot be studied as an infinite cavity: rather the boundary conditions have to be considered. The e.m. field in these cavities is well described by using the QNM theory. The lack of energy conservation gives complex, instead of real, eigenfrequencies. The evolution operator, analogous to the Hamilton operator for the conservative cases, is not Hermitian and the e.m. modes of the e.m. field are not normal but quasinormal. The importance of the QNM theory lies in the fact that it is possible to recover the orthogonal representation of the e.m. field, as is necessary to consider quantum processes.

We have applied the quasinormal mode theory to discuss the quantum problem of an atom embedded in a one-dimensional photonic band gap cavity, when it is pumped by two counterpropagating laser beams. The e.m. field is quantized in terms of the QNMs in the 1D PBG and the atom is modeled as a two-level system. In the electric dipole approximation, the atom is assumed to be weakly coupled to just one of the QNMs. This paper shows that the decay time depends on the position of the dipole inside the cavity, and can be controlled by the phase difference of the two laser beams. Such a system might therefore be relevant for a single-atom, phase-sensitive, optical memory device on the atomic scale.

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